MBF1223 | Financial Management

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L3 – Time Value of Money

Learning Objectives

1. Calculate future values and understand compounding.
2. Calculate present values and understand discounting.
3. Calculate implied interest rates and waiting time from the time value of money equation.
4. Apply the time value of money equation using formula, calculator, and spreadsheet.
5. Explain the Rule of 72, a simple estimation of doubling values.

3.1 Future Value and Compounding Interest

- The value of money at the end of the stated period is called the future or compound value of that sum of money.
  - Determine the attractiveness of alternative investments
  - Figure out the effect of inflation on the future cost of assets, such as a car or a house.

3.1 (A) The Single-Period Scenario

FV = PV + PV x interest rate, or

FV = PV(1+interest rate) ← (in decimals)

Example 1: Let's say John deposits $200 for a year in an account that pays 6% per year. At the end of the year, he will have:

FV = $200 + ($200 x .06) = $212

= $200(1.06) = $212

3.1 (B) The Multiple-Period Scenario

$$FV = PV \times (1+r)^n$$

Example 2: If John closes out his account after 3 years, how much money will he have accumulated? How much of that is the interest-on-interest component? What about after 10 years?

FV_3 = \$200(1.06)^3 = \$200*1.191016 = \$238.20,

where, 6% interest per year for 3 years = \$200 x.06 x 3=\$36

Interest on interest = \$238.20 - \$200 - \$36 =\$2.20

FV_{10} = \$200(1.06)^{10} = \$200 x 1.790847 = \$358.17

where, 6% interest per year for 10 years = \$200 x .06 x 10 = \$120

Interest on interest = \$358.17 - \$200 - \$120 = \$38.17

3.1 (C) Methods of Solving Future Value Problems

- *Method 1: The formula method*
  - Time-consuming, tedious
- *Method 2: The financial calculator approach*
  - Quick and easy
- *Method 3: The spreadsheet method*
  - Most versatile
- *Method 4: The use of Time Value tables:*
  - Easy and convenient but most limiting in scope

3.1 (C) Methods of Solving Future Value Problems (continued)

Example 3: Compounding of Interest

Let’s say you want to know how much money you will have accumulated in your bank account after 4 years, if you deposit all $5,000 of your high-school graduation gifts into an account that pays a fixed interest rate of 5% per year. You leave the money untouched for all four of your college years.

3.1 (C) Methods of Solving Future Value Problems (continued)

Example 3: Answer

Formula Method**:**

FV = PV x $(1+r)^n$ ➔ $5,000$(1.05)^4$=$6,077.53

Calculator method**:**

PV =-5,000; **N**=4; **I/Y**=5; **PMT**=0; **CPT FV**=$6077.53

Spreadsheet method:

Rate = .05; **Nper** = 4; **Pmt**=0; **PV**=-5,000; **Type** =0; **FV**=6077.53

Time value table method**:**

FV = PV(FVIF, 5%, 4) = 5000*(1.215506)=6077.53,

where (FVIF, 5%,4) = Future value interest factor listed under the 5% column and in the 4-year row of the Future Value of $1 table.

3.1 (C) Methods of Solving Future Value Problems (continued)

Example 4: Future Cost due to Inflation

Let's say that you have seen your dream house, which is currently listed at $300,000, but unfortunately, you are not in a position to buy it right away and will have to wait at least another 5 years before you will be able to afford it. If house values are appreciating at the average annual rate of inflation of 5%, how much will a similar house cost after 5 years?

3.1 (C) Methods of Solving Future Value Problems (continued)

Example 4 (Answer)

PV = current cost of the house = $300,000;

n = 5 years;

r = average annual inflation rate = 5%.

Solving for FV, we have

FV = $300,000*(1.05)(1.05)(1.05)(1.05)(1.05)

= $300,000*(1.276282)

= $382,884.5

So the house will cost $382,884.5 after 5 years

3.1 (C) Methods of Solving Future Value Problems (continued)

Calculator method:

PV =-300,000; **N**=5; **I/Y**=5; **PMT**=0; **CPT FV**=$382,884.5

Spreadsheet method:

Rate = .05; **Nper** = 5; **Pmt**=0; **PV**=-$300,000; **Type** =0; **FV**=$382,884.5

Time value table method:

FV = PV(FVIF, 5%, 5) = 300,000*(1.27628)=$382,884.5;

where (FVIF, 5%,5) = Future value interest factor listed under the 5% column and in the 5-year row of the future value of $1 table=1.276

3.2 Present Value and Discounting

- Involves discounting the interest that would have been earned over a given period at a given rate of interest.
- It is therefore the exact opposite or inverse of calculating the future value of a sum of money.
- Such calculations are useful for determining today's price or the value today of an asset or cash flow that will be received in the future.
- The formula used for determining PV is as follows:

$$PV = FV \times 1/(1+r)^n$$

3.2 (A) The Single-Period Scenario

When calculating the present or discounted value of a future lump sum to be received one period from today, we are basically deducting the interest that would have been earned on a sum of money from its future value at the given rate of interest.

i.e. $PV = FV/(1+r)$ ➔ *since* $n = 1$

So, if $FV = 100$; $r = 10\%$; *and* $n = 1$;

➔ $PV = 100/1.1 = 90.91$

3.2 (B) The Multiple-Period Scenario

When multiple periods are involved…

The formula used for determining PV is as follows:

$$PV = FV \times 1/(1+r)^n$$

where the term in brackets is the present value interest factor for the relevant rate of interest and number of periods involved, and is the reciprocal of the future value interest factor (FVIF)

3.2 Present Value and Discounting (continued)

Example 5: Discounting Interest

Let's say you just won a jackpot of $50,000 at the casino and would like to save a portion of it so as to have $40,000 to put down on a house after 5 years. Your bank pays a 6% rate of interest. How much money will you have to set aside from the jackpot winnings?

3.2 Present Value and Discounting (continued)

Example 5 (Answer)

FV = amount needed = \$40,000

N = 5 years; Interest rate = 6%;

- PV = FV x 1/ $(1+r)^n$
- PV = \$40,000 x 1/$(1.06)^5$
- PV = \$40,000 x 0.747258
- PV = \$29,890.33 ➔ Amount needed to set aside today

3.2 Present Value and Discounting (continued)

Calculator method:

FV 40,000; **N**=5; **I/Y =**6%; **PMT**=0; **CPT PV=**-$29,890.33

Spreadsheet method:

Rate = .06; **Nper** = 5; **Pmt**=0; **Fv**=$40,000; **Type** =0;

Pv=-$29,890.33

Time value table method:

PV = FV(PVIF, 6%, 5) = 40,000*(0.7473)=$29,892

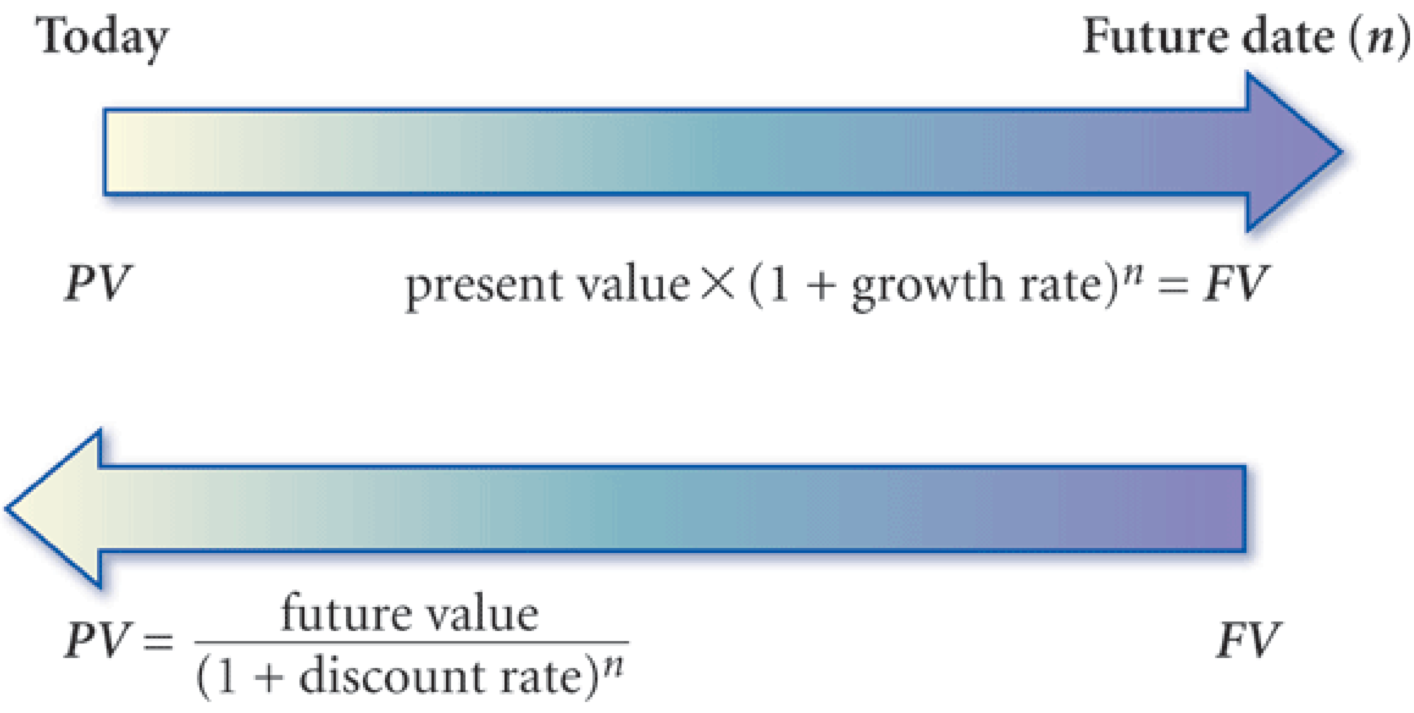
where (PVIF, 6%,5) = Present value interest factor listed under the 6% column and in the 5-year row of the Present Value of $1 table=0.7473

3.2 (C) Using Time Lines

- When solving time value of money problems, especially the ones involving multiple periods and complex combinations (which will be discussed later) it is always a good idea to draw a time line and label the cash flows, interest rates and number of periods involved.

3.2 (C) Using Time Lines (continued)

FIGURE 3.1 Time lines of growth rates (top) and discount rates (bottom) illustrate present value and future value.



3.3 One Equation and Four Variables

- Any time value problem involving lump sums—i.e., a single outflow and a single inflow—requires the use of a single equation consisting of 4 variables i.e. *PV, FV, r, n*
- If 3 out of 4 variables are given, we can solve the unknown one.

$FV = PV \times (1+r)^n$ ➔solving for future value

$PV = FV \times [1/(1+r)^n]$ ➔solving for present value

$r = [FV/PV]^{1/n} - 1$ ➔solving for unknown rate n

$= [\ln(FV/PV)/\ln(1+r)]$ ➔solving for # of periods

3.4 Applications of the Time Value of Money Equation

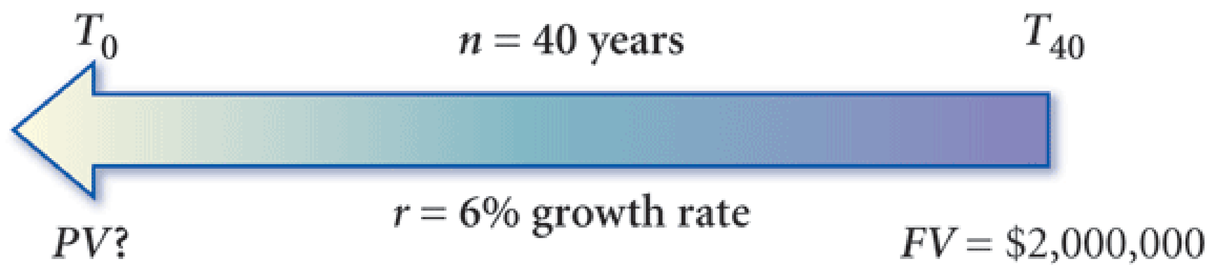
- Calculating the amount of saving required for retirement
- Determining future value of an asset
- Calculating the cost of a loan
- Calculating growth rates of cash flows
- Calculating number of periods required to reach a financial goal.

Example 3.3 Saving for retirement

EXAMPLE 3.3 **Saving for retirement (present value)**

Problem Your retirement goal is $2,000,000. The bank is offering you a certificate of deposit that is good for forty years at 6.0%. What initial deposit do you need to make today to reach your $2,000,000 goal at the end of forty years?

Solution The following time line illustrates the problem.



We designate today as T_0 and our future date forty years later as T_{40}.

Example 3.3 Saving for retirement (continued)

METHOD 1 Using the equation

$$PV = \$2{,}000{,}000 \times \frac{1}{1.06^{40}} = \$2{,}000{,}000 \times 0.0972 = \$\mathbf{194{,}444.38}$$

METHOD 2 Using the TVM keys

Input	40	6.0	?	0	2,000,000
Key	N	I/Y	PV	PMT	FV
CPT			−194,444.38		

Example 3.3 Saving for retirement (continued)

METHOD 3 Using a spreadsheet

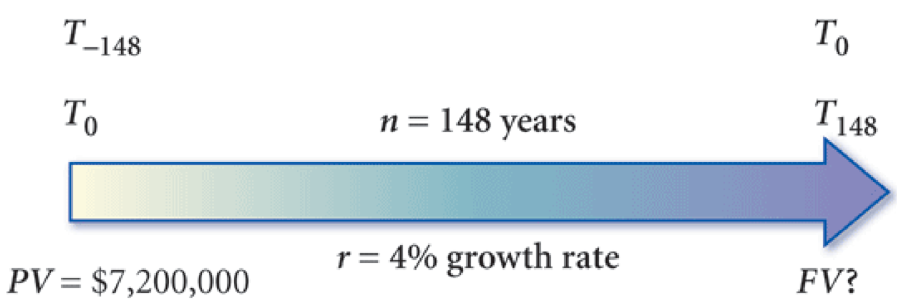
B6	*fx*	*=PV(B1,B2,B3,B4,B5)*			
Use the present value function to find the amount of dollars you need to invest today to reach $2,000,000 in 40 years.					
	A	B	C	D	E
1	Rate	0.06			
2	Nper	40			
3	Pmt	0			
4	Fv	$2,000,000.00			
5	Type	0			
6	Pv	($ 194,444.38)			

Example 3.4 Let's make a deal (future value)

EXAMPLE 3.4 Let's make a deal! (future value)

Problem In 1867, Secretary of State William H. Seward purchased Alaska from Russia for the sum of $7,200,000, or about two cents per acre. At the time, the deal was dubbed Seward's Folly, but from our vantage point today, did Seward get a bargain after all? What would it cost today if the land were in exactly the same condition as it was 148 years ago and the prevailing interest rate over this time were 4%?

Solution At first glance, it seems as if we have a present value problem, not a future value problem, but it all depends on where we are standing in reference to time. Phrasing this question another way, we could ask, "What will the value of $7,200,000 be in 148 years at an annual interest rate of 4%?" Restated this way, we can more easily view the problem as a future value problem. A time line is particularly helpful in this instance. We can show the 148-year span from T_{-148} to T_0 or from T_0 to T_{148}.



Example 3.4 Let's make a deal (continued)

METHOD 1 Using the equation

$$FV = PV \times (1 + r)^n = \$7{,}200{,}000 \times 1.04^{148}$$
$$= \$7{,}200{,}000 \times 313.8442 = \mathbf{\$2{,}389{,}278{,}156}$$

METHOD 2 Using the TVM keys

Input	148	4.0	−7,200,000	0	?
Key	N	I/Y	PV	PMT	FV
CPT					2,389,278,156

Example 3.4 Let's make a deal (continued)

METHOD 3 Using a spreadsheet

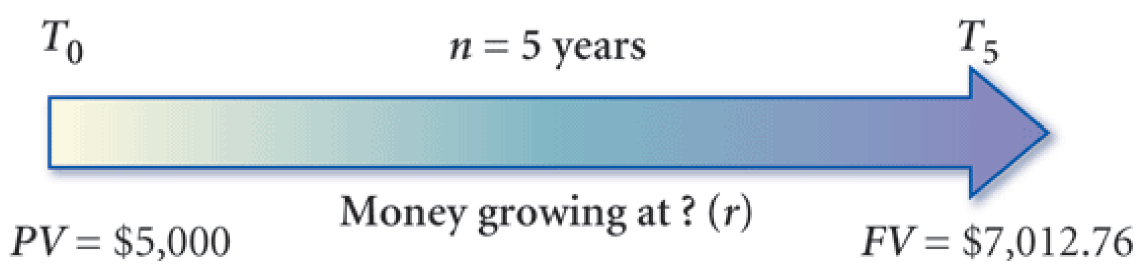
B6	*fx*	=FV(B1,B2,B3,B4,B5)			
Use the future value function to find the price of Alaska ifpurchased today instead of148 years ago.					
	A	B	C	D	E
1	Rate	0.04			
2	Nper	148			
3	Pmt	0			
4	Pv	($ 7,200,000.00)			
5	Type	0			
6	Fv	$2,389,278,156			

Example 3.5 What's the cost of that loan?

EXAMPLE 3.5 **What's the cost of that loan? (interest rate)**

Problem John, a college student, needs to borrow $5,000 today for his tuition bill. He agrees to pay back the loan in a lump-sum payment five years from now, after he is out of college. The bank states that the payment will need to be $7,012.76. If John borrows the $5,000 from the bank, what interest rate is he paying on his loan?

Solution A time line is helpful in this instance.



Example 3.5 What's the cost of that loan? (continued)

METHOD 1 Using the equation

$$r = \left(\frac{FV}{PV}\right)^{1/n} - 1 = \left(\frac{\$7{,}012.76}{\$5{,}000}\right)^{1/5} - 1$$

$$= 1.40255^{0.2} - 1 = \mathbf{0.07} \quad \text{or} \quad 7\%$$

METHOD 2 Using the TVM keys

Input	5	?	5,000	0	−7,012.76
Key	N	I/Y	PV	PMT	FV
CPT		7.00			

Example 3.5 What's the cost of that loan? (continued)

METHOD 3 Using a spreadsheet

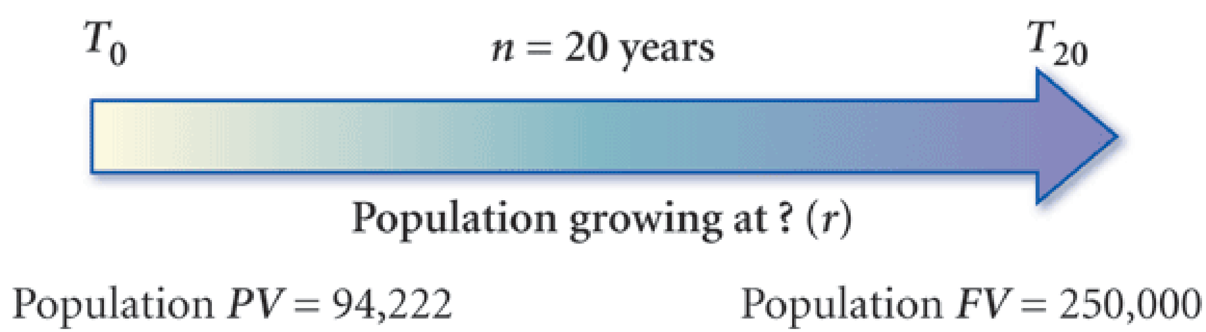
B6	*fx*	*=RATE(B1,B2,B3,B4,B5)*			
Use the rate function to see the interest rate that a loan of $5,000 is costing if it requires a payment of $7,012.76 in 5 years.					
	A	B	C	D	E
1	Nper	5			
2	Pmt	0			
3	Pv	$ 5000			
4	Fv	($7012.76)			
5	Type	0			
6	Rate	7.0%			

Example 3.6 Boomtown, USA (growth rate)

EXAMPLE 3.6 Boomtown, USA (growth rate)

Problem You are the planning commissioner for Boomtown, a growing city in the Southwest. The city council has estimated that the city's population will increase very rapidly over the next twenty years, reaching an estimated 250,000. Today the population is 94,222. What is the projected growth rate of this city?

Solution Here the present value is the current population of Boomtown: 94,222. The future value is the projected 250,000 population. The period is twenty years. See the time line.



Example 3.6 Boomtown, USA (continued)

METHOD 1 Using the equation

$$r = \left(\frac{250{,}000}{94{,}222}\right)^{1/20} - 1 = 2.6533^{1/20} - 1 = 1.05 - 1 = \mathbf{0.05} \text{ or } 5.0\%$$

METHOD 2 Using the TVM keys

Input	20	?	−94,222	0	250,000
Key	N	I/Y	PV	PMT	FV
CPT		5.0			

Example 3.6 Boomtown, USA (continued)

METHOD 3 Using a spreadsheet

B6	*fx*	=RATE(B1,B2,B3,B4,B5)

Use the rate function to find the town's growth rate if its current population is 94,222 and it will be 250,000 in 20 years.

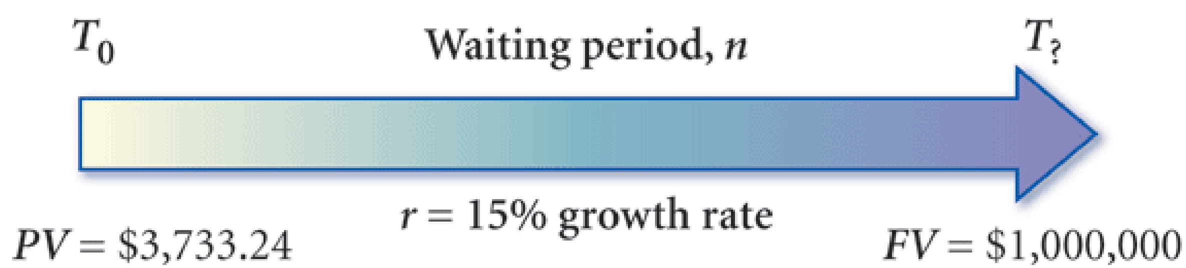
	A	B	C	D	E
1	Nper	20			
2	Pmt	0			
3	Pv	(94,222.00)			
4	Fv	250,000.00			
5	Type	0			
6	Rate	5%			

Example 3.7 When will I be rich?

EXAMPLE 3.7 When will I be rich? (waiting time)

Problem Your goal in life is to be a millionaire. Today your financial portfolio is worth $3,733.24. Having studied this chapter carefully and being a shrewd investor, you determine that you can earn 15% every year on your portfolio. You do not plan to invest any additional money in this portfolio, nor will you withdraw any funds from it before it grows to $1 million. Given your 15% interest rate, how long will you have to wait to become a millionaire if this investment represents all your wealth?

Solution See the time line.



Example 3.7 When will I be rich? (continued)

METHOD 1 Using the equation

$$n = \frac{\ln(\$1{,}000{,}000/\$3{,}733.24)}{\ln(1.15)} = \frac{\ln(267.8638)}{\ln(1.15)} = \frac{5.59}{0.1398} = \mathbf{40.00}$$

METHOD 2 Using the TVM keys

Input	?	15.0	−3,733.24	0	1,000,000
Key	N	I/Y	PV	PMT	FV
CPT	40.00				

Example 3.7 When will I be rich? (continued)

METHOD 3 Using a spreadsheet

B6	*fx*	=NPER(B1,B2,B3,B4,B5)			
Use the number of periods function to find the years it will take to grow $3,733.24 into $1 million at 15% interest.					
	A	B	C	D	E
1	Rate	0.15			
2	Pmt	0			
3	Pv	($ 3,733.24)			
4	Fv	$1,000,000.00			
5	Type	0			
6	Nper	40.00			

3.5 Doubling of Money: The Rule of 72

- The Rule of 72 estimates the number of years required to double a sum of money at a given rate of interest.
  - For example, if the rate of interest is 9%, it would take 72/9 ➔ 8 years to double a sum of money
- Can also be used to calculate the rate of interest needed to double a sum of money by a certain number of years.
  - For example, to double a sum of money in 4 years, the rate of return would have to be approximately 18% (i.e. 72/4=18).

Additional Problems with Answers
Problem 1

Joanna's Dad is looking to deposit a sum of money immediately into an account that pays an annual interest rate of 9% so that her first-year college tuition costs are provided for. Currently, the average college tuition cost is $15,000 and is expected to increase by 4% (the average annual inflation rate). Joanna just turned 5, and is expected to start college when she turns 18. How much money will Joanna's Dad have to deposit into the account?

Additional Problems with Answers
Problem 1 (Answer)

Step 1. Calculate the average annual college tuition cost when Joanna turns 18, i.e., the future compounded value of the current tuition cost at an annual increase of 4%.

PV = -15,000; n= 13; i=4%; PMT=0; CPT FV=$24, 976.10

OR

$$FV = \$15{,}000 \times (1.04)^{13} = \$15{,}000 \times 1.66507 = \$24{,}976.10$$

Additional Problems with Answers
Problem 1 (Answer) (continued)

Step 2. Calculate the present value of the annual tuition cost using an interest rate of 9% per year.

FV = 24,976.10; n=13; i=9%; PMT = 0; CPT PV = $8,146.67

(rounded to 2 decimals)

OR

PV = \$24,976.10 x (1/(1+0.09)^{13}=\$24,976.10 x 0.32618 = \$8,146.67

So, Joanna's Dad will have to deposit $8,146.67 into the account today so that she will have her first-year tuition costs provided for when she starts college at the age of 18.

Additional Problems with Answers
Problem 2

Bank A offers to pay you a lump sum of $20,000 after 5 years if you deposit $9,500 with them today. Bank B, on the other hand, says that they will pay you a lump sum of $22,000 after 5 years if you deposit $10,700 with them today. Which offer should you accept, and why?

Additional Problems with Answers
Problem 2 (Answer)

To answer this question, you have to calculate the rate of return that will be earned on each investment and accept the one that has the higher rate of return.

Bank A's Offer:

PV = -\$9,500; n=5; FV =\$20,000; PMT = 0; CPT I = 16.054%

OR

Rate = $(FV/PV)^{1/n} - 1 = (\$20,000/\$9,500)^{1/5} - 1 = 1.16054 - 1 = 16.054\%$

Additional Problems with Answers

Problem 2 (Answer) (continued)

Bank B's Offer:

PV = -$10,700; n=5; FV =$22,000; PMT = 0; CPT I = 15.507%

OR

Rate = $(FV/PV)^{1/n} - 1 = (\$22,000/\$10,700)^{1/5} - 1$

$= 1.15507 - 1 = 15.507\%$

You should accept Bank A's offer, since it provides a higher annual rate of return i.e 16.05%.

Additional Problems with Answers
Problem 3

You have decided that you will sell off your house, which is currently valued at $300,000, at a point when it appreciates in value to $450,000. If houses are appreciating at an average annual rate of 4.5% in your neighborhood, for approximately how long will you be staying in the house?

Additional Problems with Answers
Problem 3 (Answer)

PV = -300,000; FV = 450,000; I = 4.5%; PMT = 0; CPT n = 9.21 years or 9 years and 3 months

OR

n =[ln(FV/PV)]/[ln(1+i)]

n =[ln(450,000/(300,000])/[ln(1.045)]

= .40547 / .04402 = 9.21 years

Additional Problems with Answers
Problem 4

Your arch-nemesis, who happens to be an accounting major, makes the following remark, “You finance types think you know it all…well, let’s see if you can tell me, without using a financial calculator, what rate of return would an investor have to earn in order to double $100 in 6 years?” How would you respond?

Additional Problems with Answers
Problem 4 (Answer)

Use the rule of 72 to silence him once and for all, and then prove the answer by compounding a sum of money at that rate for 6 years to show him how close your response was to the actual rate of return…Then ask him politely if he would like you to be his "lifeline" on "Who Wants to be a Millionaire?"

Rate of return required to double a sum of money = 72/N = 72/6 = 12%

Verification: $\$100(1.12)^6 = \$197.38$…which is pretty close to double

Additional Problems with Answers

Problem 4 (Answer) (continued)

The accurate answer would be calculated as follows:

PV = -100; FV = 200; n = 6; PMT = 0; I = 12.246%

OR

$$r = (FV/PV)^{1/n} - 1 = (200/100)^{1/6} - 1$$

$$= 1.12246 - 1 = .12246 \text{ or } 12.246\%$$

Additional Problems with Answers
Problem 5

You want to save $25,000 for a down payment on a house. Bank A offers to pay 9.35% per year if you deposit $11,000 with them, while Bank B offers 8.25% per year if you invest $10,000 with them. How long will you have to wait to have the down payment accumulated under each option?

Additional Problems with Answers
Problem 5 (Answer)

Bank A

FV = $25,000; I = 9.35%; PMT = 0; PV = -11,000;
CPT N = 9.18 years

Bank B

FV = $25,000; I = 8.25%; PMT = 0; PV = -10,000;
CPT N = 11.558 years

Table 3.1 Annual Interest Rates at 10% for $100 Initial Deposit (Rounded to Nearest Penny)

	Beginning Balance	Accumulated Interest	Interest on Principal	Interest on Interest	Ending Balance
Year 1	$100.00	—	$10.00	—	$110.00
Year 2	$110.00	$ 10.00	$10.00	$ 1.00	$121.00
Year 3	$121.00	$ 21.00	$10.00	$ 2.10	$133.10
Year 4	$133.10	$ 33.10	$10.00	$ 3.31	$146.41
Year 5	$146.41	$ 46.41	$10.00	$ 4.64	$161.05
Year 6	$161.05	$ 61.05	$10.00	$ 6.11	$177.16
Year 7	$177.16	$ 77.16	$10.00	$ 7.71	$194.87
Year 8	$194.87	$ 94.87	$10.00	$ 9.49	$214.36
Year 9	$214.36	$114.36	$10.00	$11.43	$235.79

Table 3.2 Variable Match for Calculator and Spreadsheet

Variable	TI Calculator TVM Keys	Excel Spreadsheet Variable Names
Number of periods	N	Nper
Interest rate	I/Y (annual rate)	Rate (periodic rate)
Present value	PV	Pv
Payment	PMT	Pmt
Future value	FV	Fv

Table 3.3 Doubling Time in Years for Given Interest Rates

Interest Rate	Doubling by Rule of 72	Doubling by Equation	Difference
2%	36.00	35.00	1.00
4%	18.00	17.67	0.33
6%	12.00	11.90	0.10
8%	9.00	9.01	0.01
10%	7.20	7.27	–0.07
12%	6.00	6.12	–0.12
14%	5.14	5.29	–0.15
16%	4.50	4.67	–0.17
18%	4.00	4.18	–0.18
20%	3.60	3.80	–0.20
24%	3.00	3.22	–0.22
30%	2.40	2.64	–0.24